DSP Problem set 1

1. Given the function x(t) = 12e-13*cos(3πt+ π/8), where t is in seconds and x is in Telsa, state the following including units.

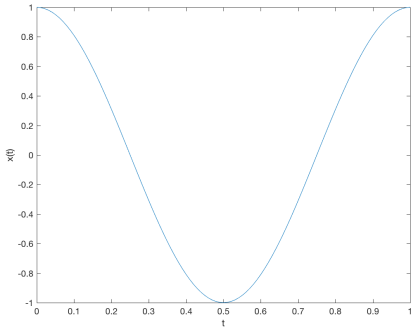
(a) Amplitude: **12e-13 T**

(b) Period: **2/3 sec**

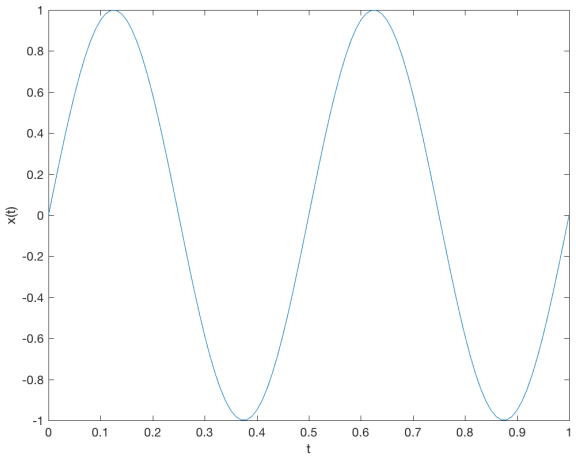
(c) Phase shift: **π/8 rad**

2. Draw the following functions.

(a) x(t) = cos(2πt)

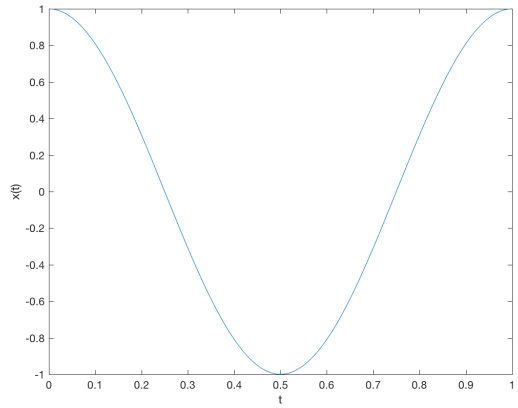


(b) x(t) = sin(4πt)

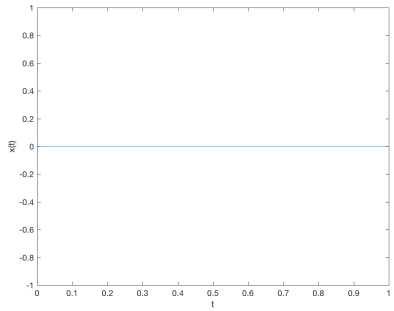


Or 1 cycle with half the domain

(c) $x(t) = \sin(2\pi t + \pi/2)$



(d) $x(t) = \sin(2n\pi t) + \sin(2n\pi - t)$, where $-t$ indicates time reversal and n is and integer.



3. Solve the following integrals.

(a) $\int_0^1 \sin(2\pi t)\, dt$

0

(b) $\int_0^{2\pi} \sin(t)\, dt$

0

(c) $\int_{-\infty}^{\infty} \sin(4\pi t) * \sin(6\pi t)\, dt$

0

4. Are the following functions periodic? If so, state their period

(a) $x(t) = \frac{e^{\frac{i\pi t}{3}} + e^{-\frac{i\pi t}{3}}}{2} + \frac{e^{\frac{i\pi t}{4}} - e^{-\frac{i\pi t}{4}}}{2i}$

x(t) = cos(pi/3)+sin(pi/4).
Frequencies: (pi/3)/2pi = 1/6 Hz, (pi/4)/2pi = 1/8 Hz
Periods: 6 sec, 8sec
LCM(6,8) = 24, 24 sec

(b) $x(t) = \sin(2t) + \sin(3\pi t)$

Frequencies: 2/2pi = 1/pi Hz, 3pi/2pi = 3/2 Hz
Periods: pi sec, 2/3 sec
LCM(pi,2/3) not integer, not periodic

(c) $x(t) = 0.3\sin(2\pi t + \pi) + \sqrt{2}\cos(3\pi t - \pi/4) + 4\sin(5\pi t + \pi/3)$

Frequencies: 1, 3/2, 5/2
Periods: 1, 2/3, 2/5
LCM(1,2/3, 2/5) = LCM(15/15, 10/15, 6/15) = 30/15= 2 sec

(d) (d) $x(t) = \sin(7\pi t) + \cos\left(\frac{1}{13}\pi t\right)$

Frequencies: 7pi/2pi = 7/2,(pi/13) / (1/2 pi) = 13/2
Periods: 2/7, 1/26
LCM(2/7, 13/2) = LCM(4/14,91/14) = 364/14 = 26 sec

5. I am interested in interactions between the theta (4-7 Hz) and high gamma (60-120Hz) waveforms in my event-related LFP signal. Please help me figure out my signal acquisition parameters. (show you work)

(a) How fast to I need to acquire samples?

2*120 = 240 Hz (minimum)

(b) How long do my epochs (data segments) need to be?

1/T = 4; = 0.25 seconds

(c) If I want to subtract the pre-stimulus baseline from the PSD how long do my epochs need to be?

0.25 sec prestim + 0.25 sec poststim = 0.5 seconds

(d) I hypothesize that the theta band in actuality 4.000-6.243 Hz. How long must my epochs test this hypothesis? What must my sampling rate be?

1/.001 = 1000 seconds; the sampling rate is inconsequential

(e) I notice my animal is switching between two behaviors every 200 ms. I predict that peak frequency differences either of my two bands might be driving this. What is the smallest high-gamma-band peak frequency difference I can resolve in the inter-behavioral-change-period? The theta-band frequency difference?

200ms = 1/5 sec, (1/5)^-1 = 5 Hz differences. The frequency resolution is the same for all frequencies, including theta, but note that lowest non-zero resolvable frequency is 5 Hz.

6. See Power Spectral Density matlab demo:

(a) What is the Nyquist frequency of the simulated signal, with given parameters.

2.5 Hz

(b) Name 3 differences between the `fft` and `pwelch` PSD.

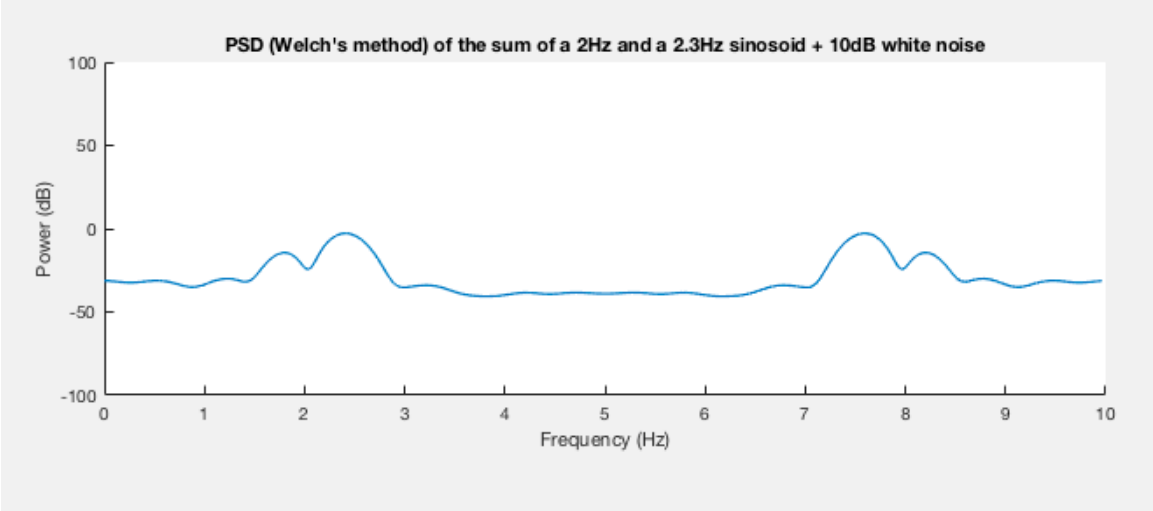
'The pwelch PSD has less variance'
'The pwelch peak frequencies are less well defined / broader'
'The pwelch peaks are a bit attenuated relative to the fft'

(c) What is the effect of decreasing the window size, thereby increasing the number of windows?

'The pwelch PSD gets Welchier: smoother, but less defined, eventually the'
'2 Hz and 2.3 hertz peaks combine'

(d) What is the smallest window (in samples) capable of resolving two separate component frequencies. Plot the PSD.

'33 samples'



(bonus) Assuming 50% overlap how many windows are there as a function of N and the even window length, n ?

'n_win = N-n/2'